# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **PHYSICS**

FOURTH SEMESTER – **APRIL 2023**

**UMT 4402 – MATHEMATICS FOR PHYSICS - II**

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

## PART - A

**Answer ALL questions.** **(10 x 2 = 20 marks)**

1. Define odd and even functions.
2. Obtain the Fourier coefficient $a_0$ for the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval 0 to $2\pi$.
3. Given a real-life situation that can be transformed into a differential equation.
4. Prove that $(a^2 - 2xy - y^2)dx - (x + y)^2 dy = 0$ is an exact equation.
5. Solve: $(D^2 + 5D + 4)y = 0$
6. Obtain the particular solution of $(D^2 + 2D + 1)y = e^{2x}$.
7. Find $L(t^2 + 2t)$.
8. Find $L^{-1}\left(\frac{1}{s^2+4}\right)$.
9. When do you say that a vector is irrotational?
10. State Gauss divergence theorem.

## PART - B

**Answer any FIVE questions.** **(5 x 8 = 40 marks)**

11. Find a sine series expansion of $f(x) = c$ in the range 0 to $\pi$.
12. Solve: $\frac{dy}{dx} + y\,cos\,x = \frac{1}{2} sin\,2\,x$
13. Use the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = tan2x$.
14. Find $L(te^{-t}sint)$.
15. Evaluate $\iint_S \vec{A} \cdot \hat{n}\, dS$ if $\vec{A} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and $S$ is the surface of $2x + y + 2z = 6$ in the first octant.
16. Express $f(x) = x, -\pi < x < \pi$ as a Fourier expansion.
17. Solve: $x\sqrt{1 + y^2}dx + y\sqrt{1 + x^2}\frac{dy}{dx} = 0$
18. Find the directional derivative of $\Phi(x, y, z) = 3x^2 + 2y - 3z$ at the point (1, 1, 1) in the direction specified by $2\vec{i} + 2\vec{j} - \vec{k}$.

## PART - C

**Answer any TWO questions.** **(2 x 20 = 40 marks)**

19. Express $f(x) = x^2$ as Fourier series with period $2\pi$ to be valid in the interval $-\pi$ to $\pi$. Hence deduce that (i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{6}$, (ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots = \frac{\pi^2}{12}$.

20. a) Solve: $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

b) Solve: $(D^2 + 4D + 4)y = e^x + \cos 2x$

21. a) Obtain the inverse Laplace transform of $\frac{1}{(s^2+4s+5)}$.

b) Using Laplace transform, solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$, $y = \frac{dy}{dt} = 0$ when $t = 0$.

22. Verify Green's theorem in the $XY$ plane for $\int_C \{(3x - 8y^2)dx + (4y - 6xy)dy\}$ where $C$ is the boundary of the region given that $x = 0, y = 0, x + y = 1$.

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